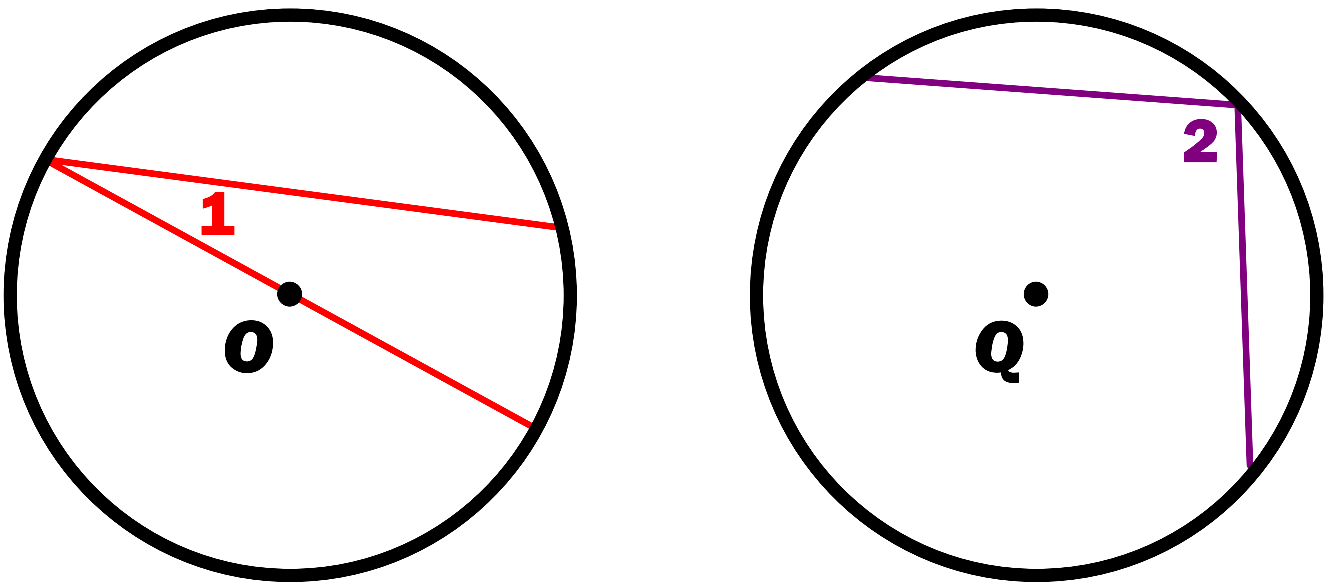
GEOMETRY - LESSON 12D
INSCRIBED ANGLES

INSCRIBED ANGLE: an angle whose vertex is on a circle and whose sides contain chords of the circle



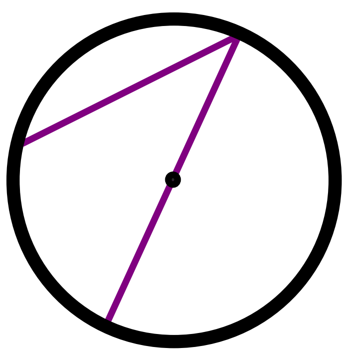
$\angle 1$ and $\angle 2$ are inscribed angles.

THE MEASURE OF AN INSCRIBED ANGLE

THEOREM

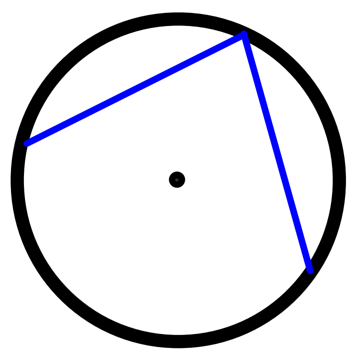
The measure of an inscribed angle is equal to half the measure of its intercepted arc.

The conditions of this theorem can be interpreted in multiple ways. For this reason, we will need several proofs representing the varied possible cases.



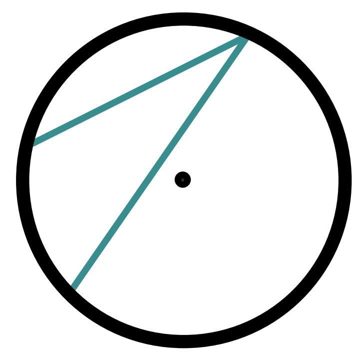
<u>Case 1</u>

The center of the circle lies ON the angle.



<u>Case 2</u>

The center of the circle lies INSIDE the angle.



<u>Case 3</u>

The center of the circle lies OUTSIDE the angle.

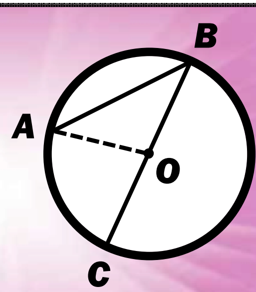
THE MEASURE OF AN INSCRIBED ANGLE

THEOREM

The measure of an inscribed angle is equal to half the measure of its intercepted arc.

GIVEN: $\angle ABC$ inscribed in $\odot O$

PROVE: $m\angle ABC = \frac{1}{2} m\overset{\frown}{AC}$



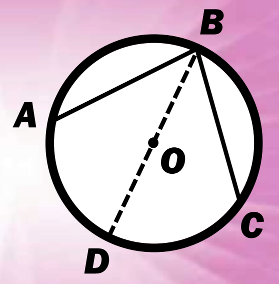
(CASE 1 - CENTER ON ANGLE)

STATEMENTS	***REASONS***
1. Draw radius $\overline{OA}$	1. Auxiliary segment
2. $\overline{OA} \cong \overline{OB}$	2. All radii of a circle are $\cong$
3. $\angle OAB \cong \angle ABC$ or $m\angle OAB = m\angle ABC$	3. In a $\triangle$, 2 $\cong$ sides ⇒ 2 $\cong \angle$s
4. $m\angle ABC + m\angle OAB = m\angle AOC$	4. Measure of exterior $\angle$ is equal to sum of remote interior $\angle$s
5. $m\angle ABC + m\angle ABC = m\angle AOC$	5. Substitution (3 , 4)
6. $2m\angle ABC = m\angle AOC$	6. Simplifying Terms
7. $m\overset{\frown}{AC} = m\angle AOC$	7. The measure of a minor arc is = to the measure of its central $\angle$ (definition of measure of minor arc)
8. $2m\angle ABC = m\overset{\frown}{AC}$	8. Substitution (7 , 6)
9. $m\angle ABC = \frac{1}{2} m\overset{\frown}{AC}$	9. Division Property of =

THE MEASURE OF AN INSCRIBED ANGLE

THEOREM

The measure of an inscribed angle is equal to half the measure of its intercepted arc.



GIVEN: $\angle ABC$ inscribed in $\odot O$

PROVE: $m\angle ABC = \frac{1}{2} m\overset{\frown}{AC}$

(CASE 2 - CENTER INSIDE ANGLE)

STATEMENTS	***REASONS***
1. Draw diameter $\overline{BD}$	1. Auxiliary segment
2. $m\angle ABC = m\angle ABD + m\angle DBC$	2. Angle Addition Postulate
3. $m\angle ABD = \frac{1}{2} m\overset{\frown}{AD}$ $m\angle DBC = \frac{1}{2} m\overset{\frown}{DC}$	3. Result proven in Case 1
4. $m\angle ABC = \frac{1}{2} m\overset{\frown}{AD} + \frac{1}{2} m\overset{\frown}{DC}$	4. Substitution (3 , 2)
5. $2m\angle ABC = m\overset{\frown}{AD} + m\overset{\frown}{DC}$	5. Multiplication Property of =
6. $m\overset{\frown}{AC} = m\overset{\frown}{AD} + m\overset{\frown}{DC}$	6. Arc Addition Postulate
7. $2m\angle ABC = m\overset{\frown}{AC}$	7. Substitution (6 , 5)
8. $m\angle ABC = \frac{1}{2} m\overset{\frown}{AC}$	8. Division Property of =

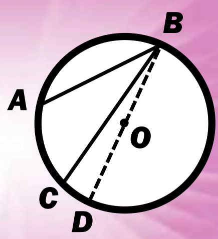
THE MEASURE OF AN INSCRIBED ANGLE

THEOREM

The measure of an inscribed angle is equal to half the measure of its intercepted arc.

GIVEN: $\angle ABC$ inscribed in $\odot O$

PROVE: $m\angle ABC = \frac{1}{2} m\overset{\frown}{AC}$



(CASE 3 - CENTER OUTSIDE ANGLE)

STATEMENTS	REASONS
1. Draw diameter $\overline{BD}$	1. Auxiliary segment
2. $m\angle ABC + m\angle DBC = m\angle ABD$	2. Angle Addition Postulate
3. $m\angle DBC = m\angle DBC$	3. Reflexive Property
4. $m\angle ABC = m\angle ABD - m\angle DBC$	4. Subtraction Property of = (2 - 3)
5. $m\angle ABD = \frac{1}{2} m\overset{\frown}{AD}$ $m\angle DBC = \frac{1}{2} m\overset{\frown}{DC}$	5. Result proven in Case 1
6. $m\angle ABC = \frac{1}{2} m\overset{\frown}{AD} - \frac{1}{2} m\overset{\frown}{DC}$	6. Substitution (5 , 4)
7. $2m\angle ABC = m\overset{\frown}{AD} - m\overset{\frown}{DC}$	7. Multiplication Property of =
8. $m\overset{\frown}{AC} + m\overset{\frown}{DC} = m\overset{\frown}{AD}$	8. Arc Addition Postulate
9. $m\overset{\frown}{DC} = m\overset{\frown}{DC}$	9. Reflexive Property
10. $m\overset{\frown}{AC} = m\overset{\frown}{AD} - m\overset{\frown}{DC}$	10. Subtraction Property of = (8 - 9)
11. $2m\angle ABC = m\overset{\frown}{AC}$	11. Substitution (10 , 7)
12. $m\angle ABC = \frac{1}{2} m\overset{\frown}{AC}$	12. Division Property of =

INSCRIBED ANGLES AND INTERCEPTED ARCS

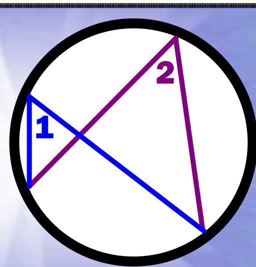
THEOREM ***(Now proven to be true in all cases)***

The measure of an inscribed angle is equal to half the measure of its intercepted arc.

COROLLARY 1

If 2 inscribed angles intercept the same arc, then the angles are congruent.

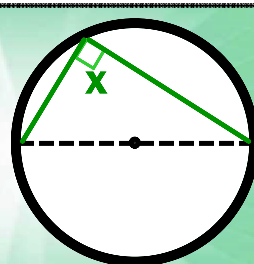
$\angle 1 \cong \angle 2$



COROLLARY 2

An angle inscribed in a semicircle is a right angle.

$\angle$ X is a right angle

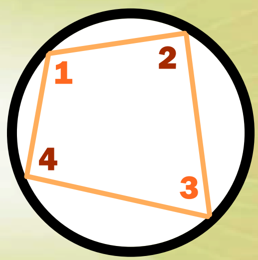


COROLLARY 3

If a quadrilateral is inscribed in a circle, then its opposite angles are supplementary.

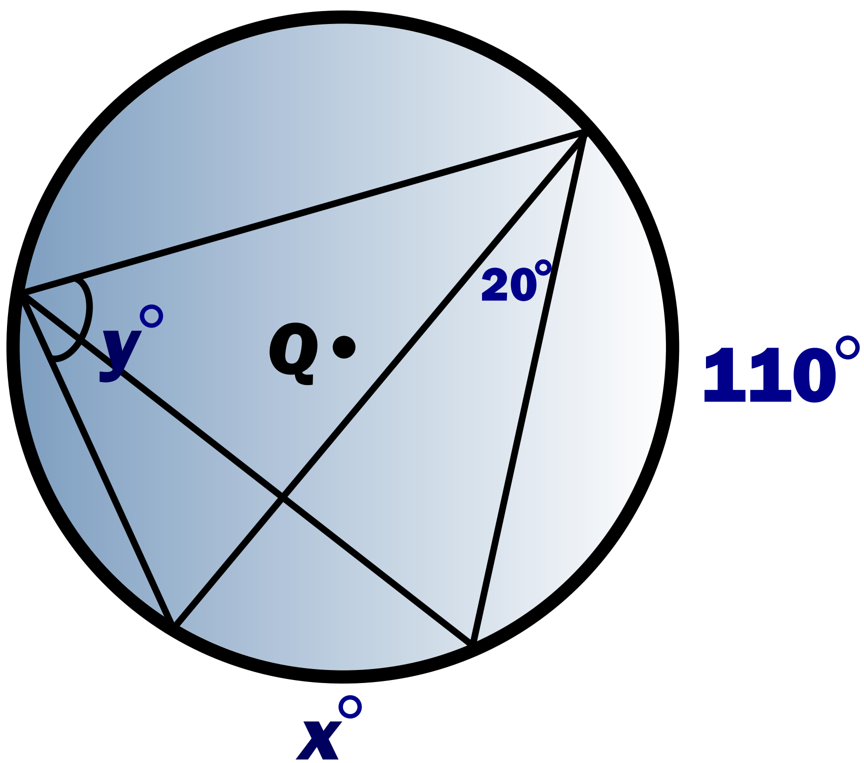
$\angle 1$ is supplementary to $\angle 3$

$\angle 2$ is supplementary to $\angle 4$



INSCRIBED ANGLES AND INTERCEPTED ARCS

ex1) **Find the values of *x* and *y*.**



$$\frac{x}{2} = 20$$

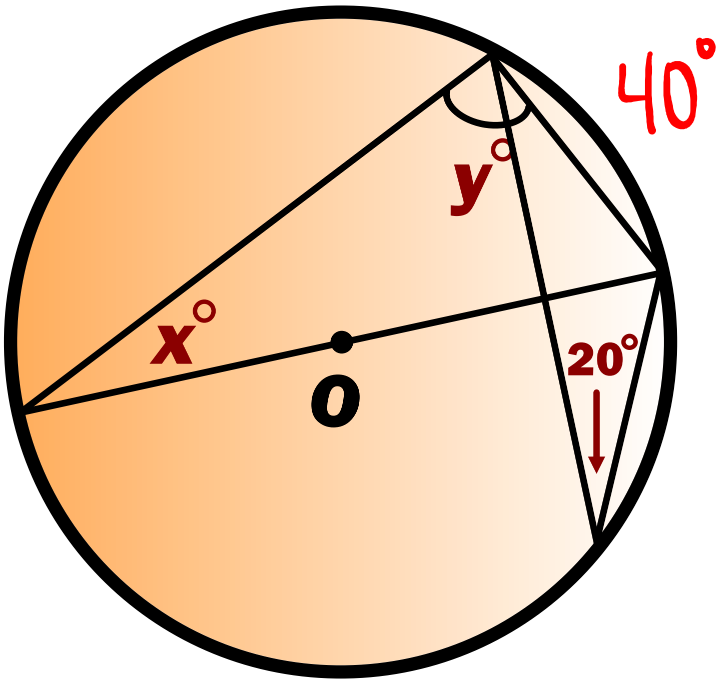
$$\boxed{x = 40}$$

$$y = \frac{110 + 40}{2}$$

$$\boxed{y = 75}$$

INSCRIBED ANGLES AND INTERCEPTED ARCS

ex2) Find the values of *x* and *y*.



$x = \frac{40}{2}$

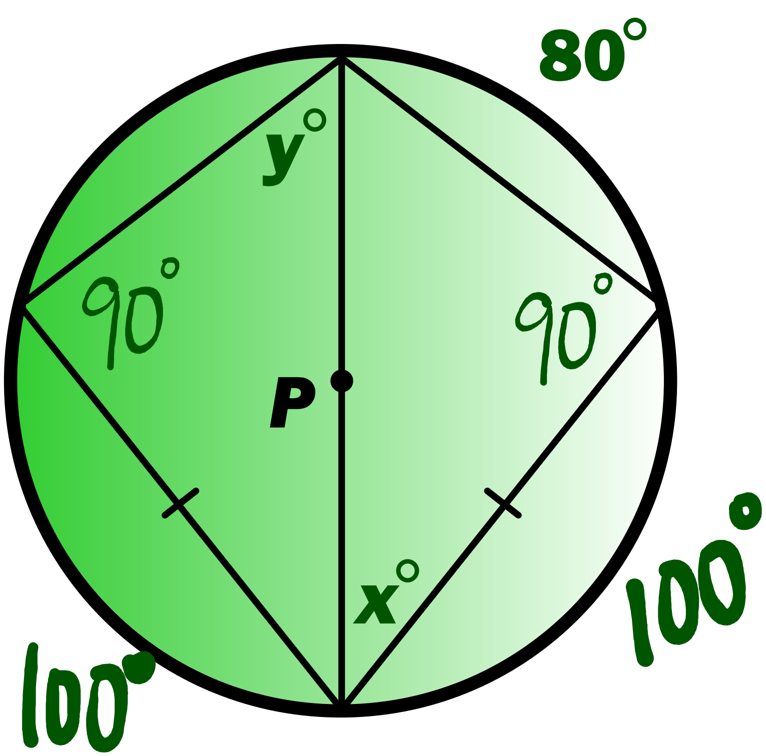
$x = 20$

y inscribed in semicircle:

$y = 90$

INSCRIBED ANGLES AND INTERCEPTED ARCS

ex3) Find the values of *x* and *y*.



$x = \frac{80}{2}$

$x = 40$

$y = \frac{100}{2}$

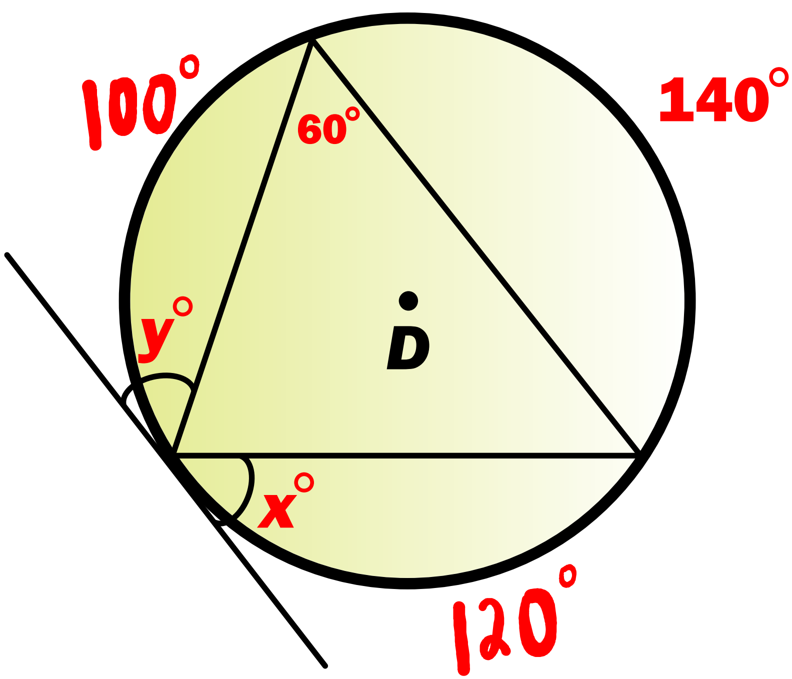
$y = 50$

ANGLE FORMED BY A CHORD AND A TANGENT

THEOREM ***(PROOF IN 3 CASES FOUND IN TEXTBOOK)***

The measure of an angle formed by a chord and a tangent is equal to half the measure of its intercepted arc.

ex4) **Find the values of *x* and *y*.**



$x = \frac{120}{2}$

$x = 60$

$y = \frac{100}{2}$

$y = 50$