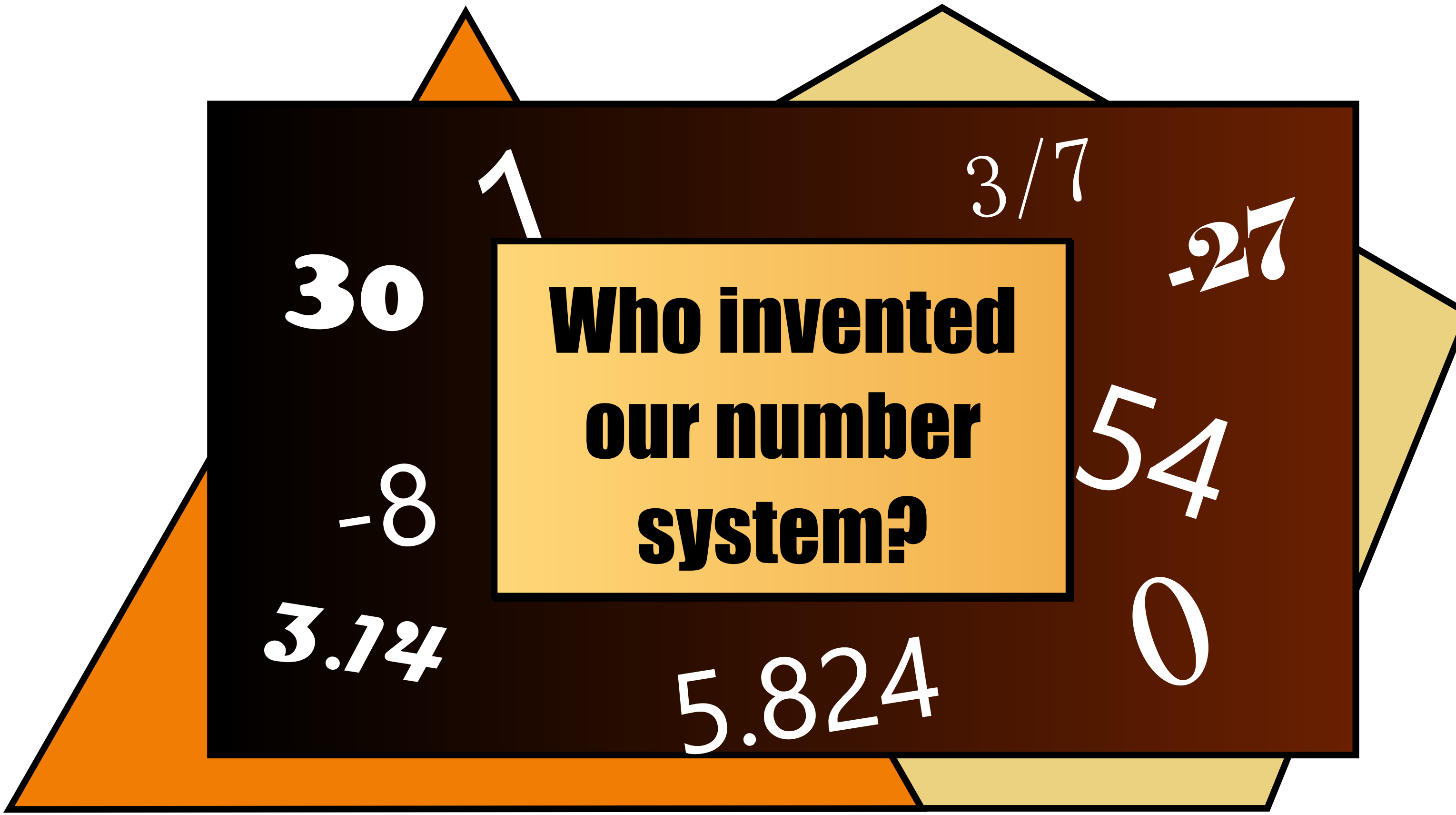
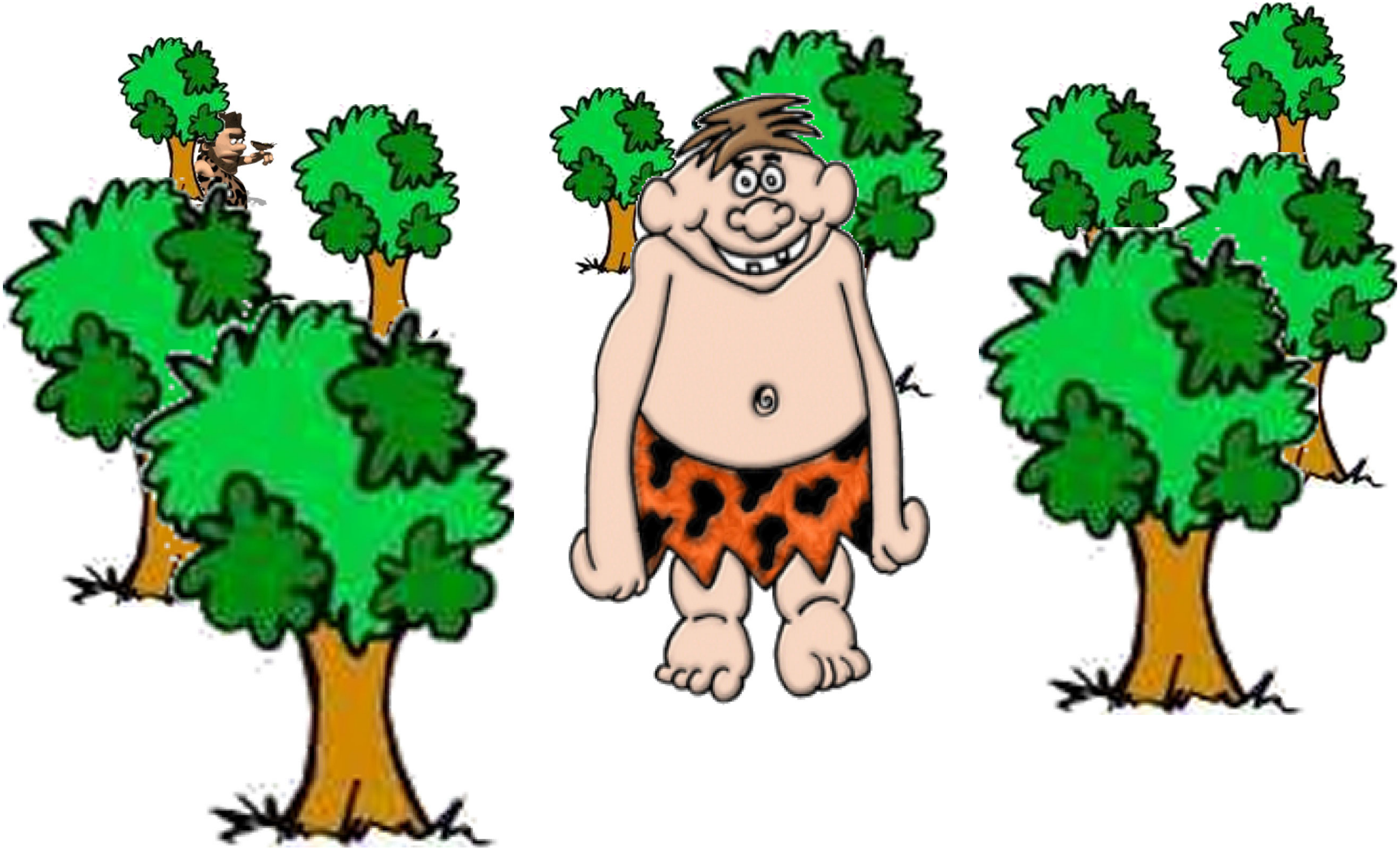
PRINCIPLES - LESSON 1F
SUBSETS OF THE REAL NUMBERS



Our number system was invented by a caveman named Og. (maybe)

OG THE CAVEMAN



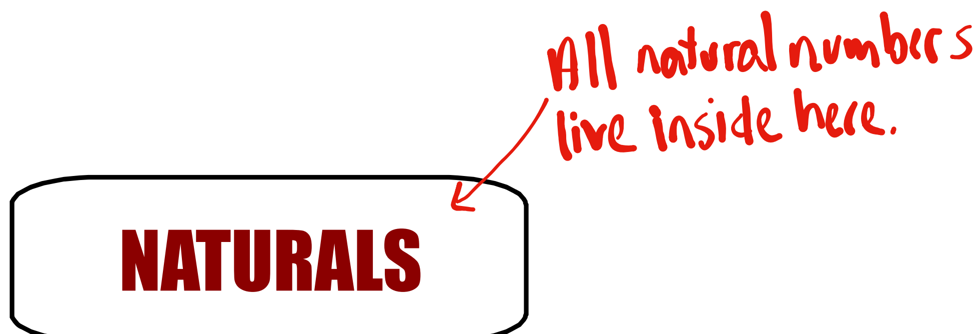
THE FIRST NUMBER SYSTEM

Og counting trees

NATURAL NUMBERS: { 1, 2, 3, 4, 5, 6, 7, . . . }

THE NUMBER SYSTEM SO FAR

Let's draw a picture of our number system!
We will add to this diagram as we go.



NOG THE CAVEMAN

THE NUMBER SYSTEM EXPANDS

Nog has no trees. We need a new number.
Zero is invented.

NATURAL NUMBERS: { 1, 2, 3, 4, 5, 6, 7, . . . }

WHOLE NUMBERS: { 0, 1, 2, 3, 4, 5, 6, 7, . . . }

NOTE: I made up everything about Og and Nog.

THE NUMBER SYSTEM SO FAR

CLOSURE PROPERTY

Closure Property: when you combine any two elements of a set, the result is also included in the set

ex1) Are the whole numbers closed under addition?

Yes! Adding 2 whole numbers will always result in another whole number.

ex2) Are the whole numbers closed under multiplication?

Yes! Multiplying 2 whole numbers will always result in another whole number.

ex3) Are the whole numbers closed under subtraction?

No! ex) $5 - 8 = -3$ ← not a whole number

THE NUMBER SYSTEM EXPANDS

In the whole number system, 4 - 7 has no answer.
The number system had to expand to include negative numbers.

NATURAL NUMBERS: { 1, 2, 3, 4, 5, 6, 7, . . . }

WHOLE NUMBERS: { 0, 1, 2, 3, 4, 5, 6, 7, . . . }

INTEGERS: { . . . -3, -2, -1, 0, 1, 2, 3 . . . }

INTEGERS = positive wholes, negative wholes, and zero
Integers are pretty numbers (not fractions or decimals).

THE NUMBER SYSTEM SO FAR

ARE THE INTEGERS CLOSED?

Closure Property: when you combine any two elements of a set, the result is also included in the set

ex4) Are the integers closed under addition?

Yes! Adding 2 integers will always result in another integer.

ex5) Are the integers closed under multiplication?

Yes! Multiplying 2 integers will always result in another integer.

ex6) Are the integers closed under subtraction?

Yes! Subtracting 2 integers will always result in another integer.

ex7) Are the integers closed under division?

No! ex) $3 \div 4 = 0.75$ ← not an integer

THE NUMBER SYSTEM EXPANDS

In the integers, $3 \div 4$ has no answer.
The number system had to expand to include fractions and decimals.

NATURAL NUMBERS: $\{1, 2, 3, 4, 5, 6, 7, \ldots\}$

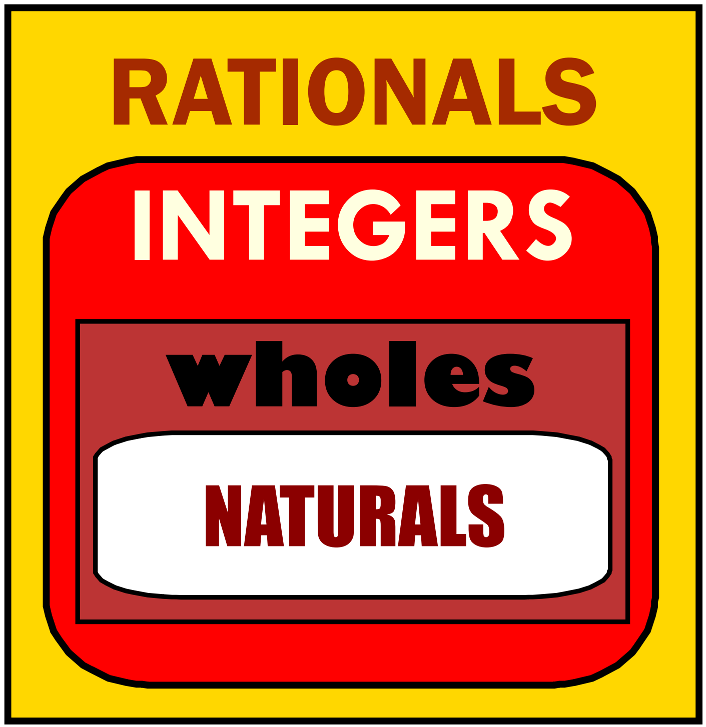
WHOLE NUMBERS: $\{0, 1, 2, 3, 4, 5, 6, 7, \ldots\}$

INTEGERS: $\{\ldots -3, -2, -1, 0, 1, 2, 3 \ldots\}$

RATIONALS: $\{\ldots -2, -\frac{3}{4}, 0, 1\frac{1}{2}, 2.38, 3 \ldots\}$

RATIONAL NUMBER: any number that CAN BE written as a quotient of two integers

THE NUMBER SYSTEM SO FAR



RATIONAL NUMBERS

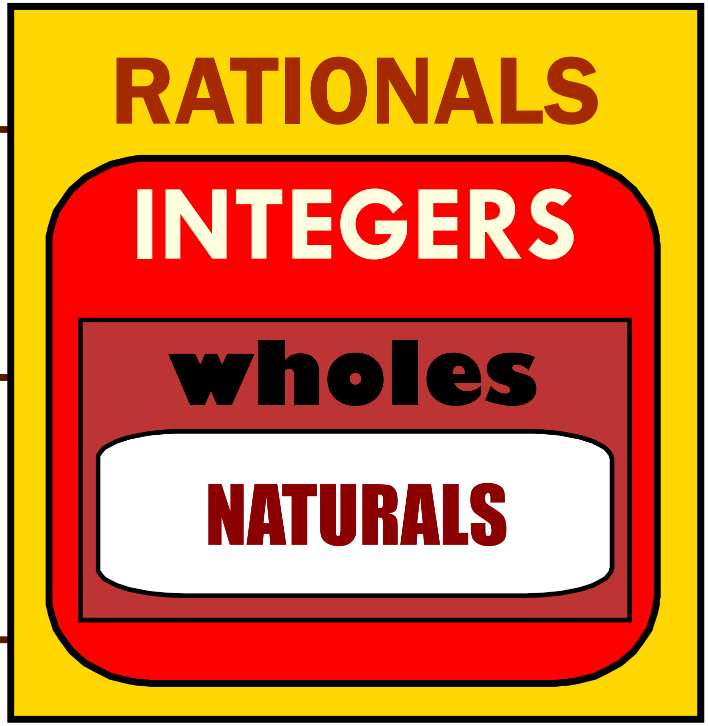
Show that the following numbers are rational. (just write them as fractions)

ex8) 4 $= \frac{4}{1}$ (or $\frac{8}{2}$ or $\frac{40}{10}$ ...)

ex9) -2 $= \frac{-2}{1}$

ex10) 0 $= \frac{0}{1}$

ex11) 0.5 $= \frac{5}{10}$ (or $\frac{1}{2}$)



ARE THE RATIONALS CLOSED?

Closure Property: when you combine any two elements of a set, the result is also included in the set

ex12) Are the rationals closed under addition?

Yes! Adding 2 rationals will always result in another rational.

ex13) Are the rationals closed under multiplication?

Yes! Multiplying 2 rationals will always result in another rational.

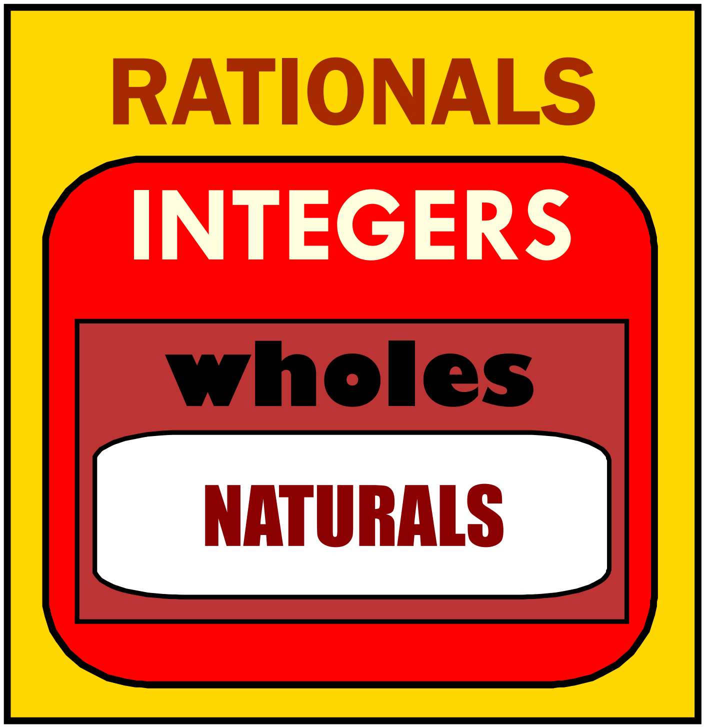
ex14) Are the rationals closed under subtraction?

Yes! Subtracting 2 rationals will always result in another rational.

ex15) Are the rationals closed under division?

Yes! Dividing 2 rationals will always result in another rational.

WHEN RATIONAL NUMBERS ARE WRITTEN AS DECIMALS



When a rational number is written as a decimal it will do one of 2 things:

1. Terminate (end)

2. Repeat

CAN ALL NUMBERS BE WRITTEN AS FRACTIONS?

WHAT KIND OF NUMBERS CANNOT BE WRITTEN AS A FRACTION?

IRRATIONAL NUMBERS

IRRATIONAL NUMBER: any number that CANNOT BE written as a quotient of two integers

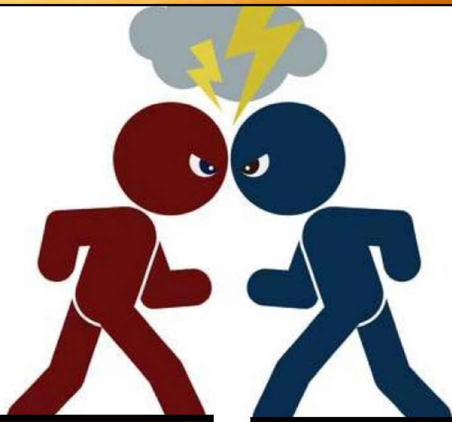
ex16) List 3 irrational numbers.

π , $\sqrt{2}$, $\sqrt{11}$, $\sqrt{19}$ ← an extra irrational thrown in for fun! (a bonus irrational)

Any number containing π is irrational.

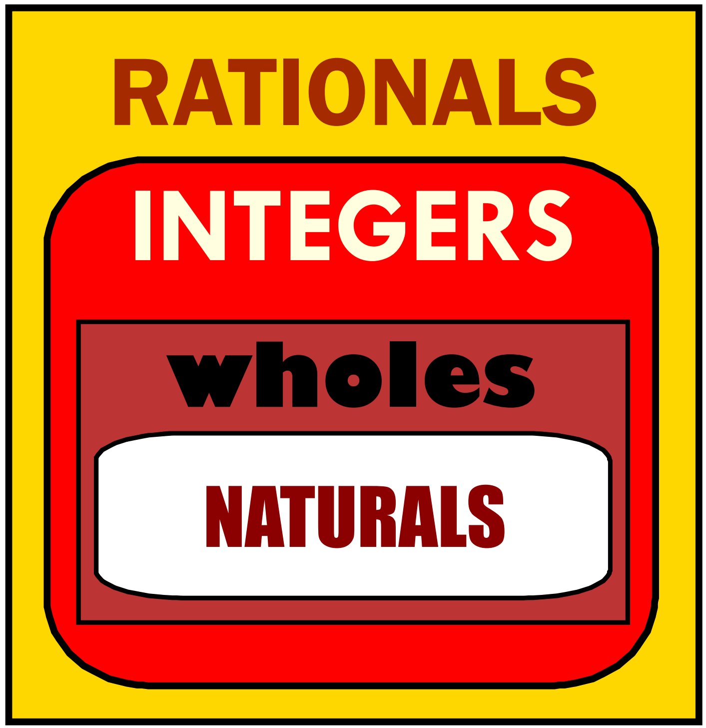
Any square root that does not work out nicely is irrational.

RATIONAL VS IRRATIONAL

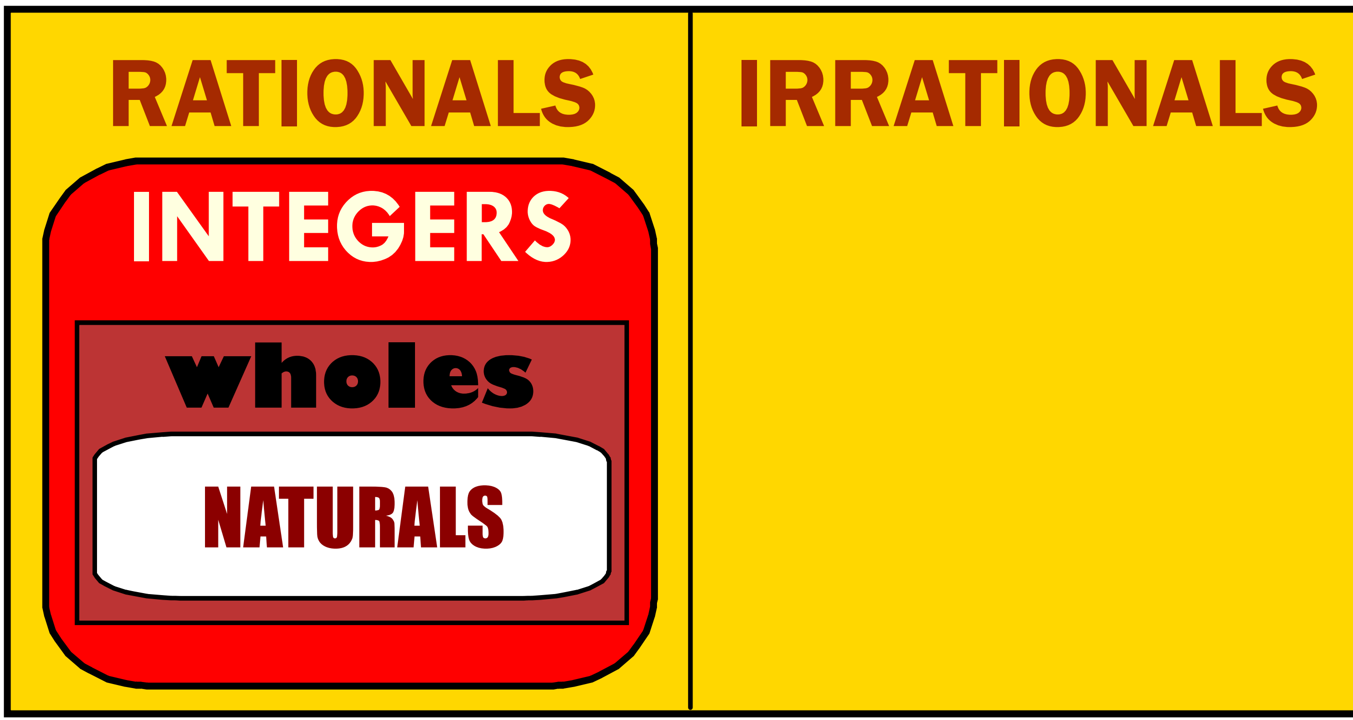


RATIONAL NUMBERS	IRRATIONAL NUMBERS
CAN be written as a fraction	**CANNOT** be written as a fraction
As a decimal: will **TERMINATE** or **REPEAT**	As a decimal: will **CONTINUE FOREVER WITHOUT REPEATING**

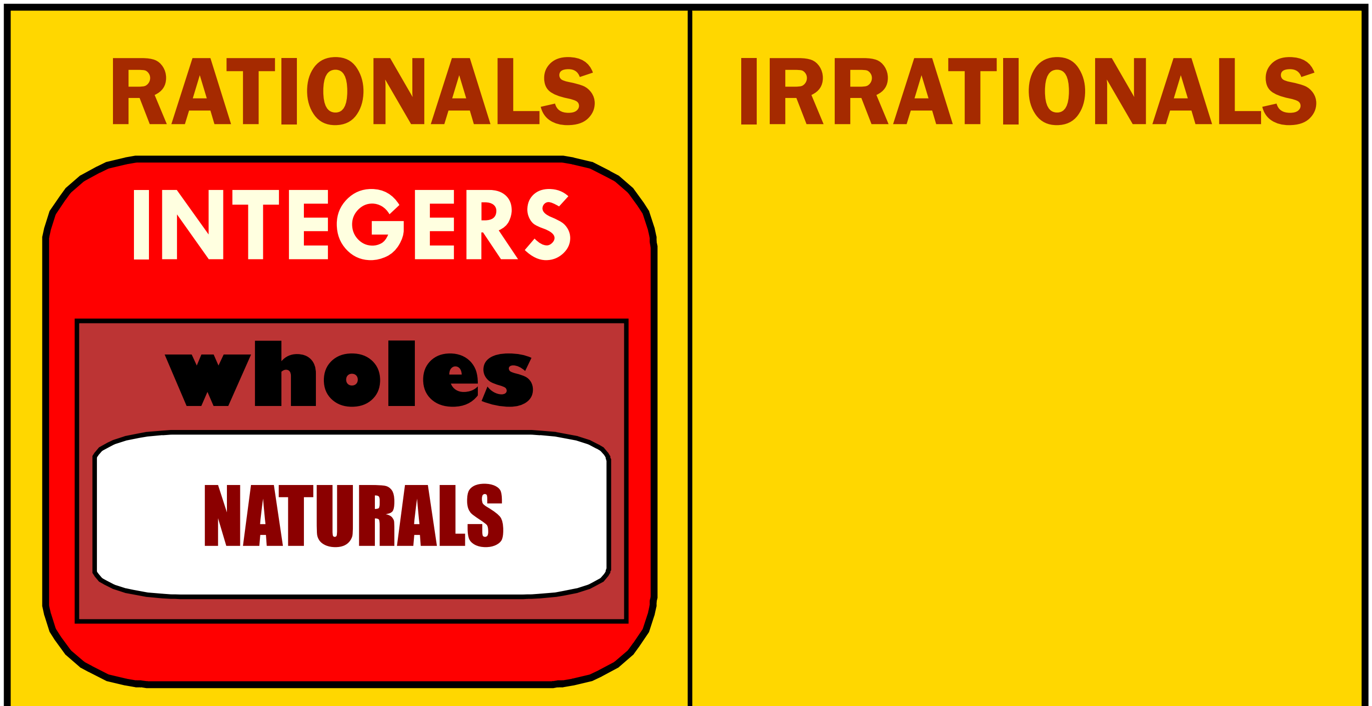
WHERE IN OUR DIAGRAM DO THE IRRATIONALS BELONG?



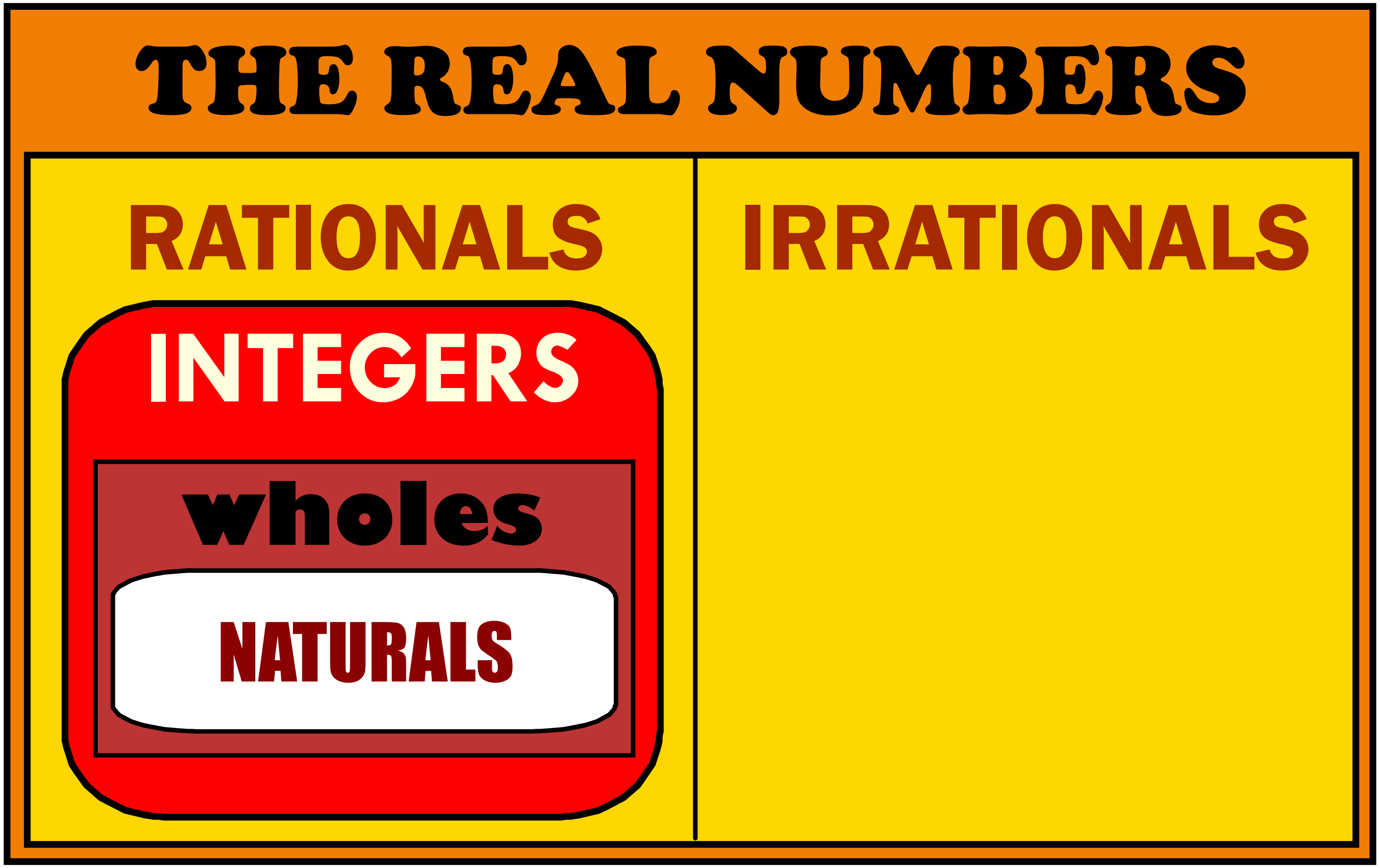
THE NUMBER SYSTEM SO FAR



WHERE IN OUR DIAGRAM DO THE REAL NUMBERS BELONG?



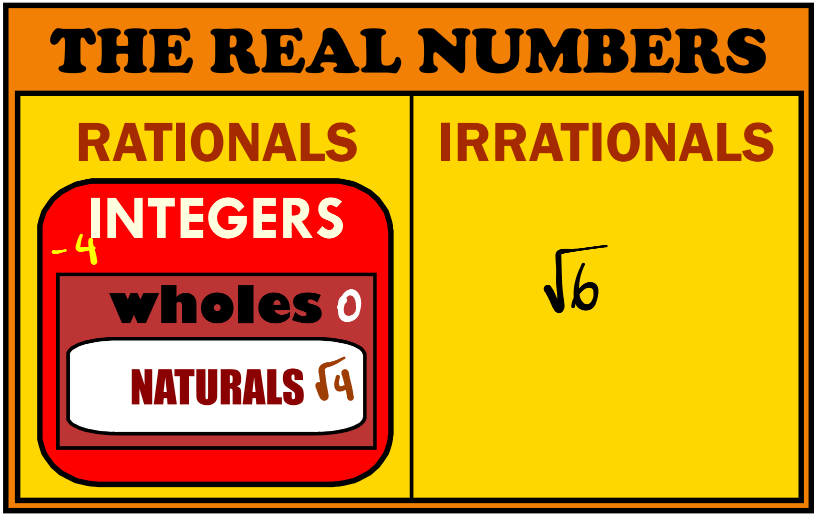
THE REAL NUMBERS



REAL NUMBER: for our purposes, every number is a real number

SUBSETS OF THE REALS

To which subsets of the reals does each belong?



ex17) -4 integer, rational, real

ex18) 0 whole, integer, rational, real

ex19) $\sqrt{6}$ irrational, real

ex20) $\sqrt{4}$ ← $\sqrt{4} = 2$ natural, whole, integer, rational, real

Abbreviations:

natural: N   whole: W   integer: Z

rational: Q   irrational: I or Ir   real: R

(some say that there is no technical abbreviation for irrational)