p. 413, #21-30 all

21. $5n^3 - 30n^2 + 40n = 0$

$5n(n^2 - 6n + 8) = 0$

$5n(n - 2)(n - 4) = 0$

$5n = 0 \quad or \quad n - 2 = 0 \quad or \quad n - 4 = 0$

$\frac{5n}{5} = \frac{0}{5}$ $\qquad$ $\underline{+2} \ \underline{+2}$ $\qquad$ $\underline{+4} \ \underline{+4}$

$n = 0 \qquad n = 2 \qquad n = 4$

The roots are $n = 0$, $n = 2$, and $n = 4$.

22. $k^4 - 100k^2 = 0$

$k^2(k^2 - 100) = 0$

$k^2(k^2 - 10^2) = 0$

$k^2(k + 10)(k - 10) = 0$

$k^2 = 0 \quad or \quad k + 10 = 0 \quad or \quad k - 10 = 0$

$\underline{-10} \ \underline{-10}$ $\qquad$ $\underline{+10} \ \underline{+10}$

$k = 0 \qquad k = -10 \qquad k = 10$

The equation has roots of $k = -10$, $k = 10$, and repeated roots of $k = 0$.

23.

$$x^3 + x^2 = 4x + 4$$
$$x^3 + x^2 - 4x = 4x - 4x + 4$$
$$x^3 + x^2 - 4x = 4$$
$$x^3 + x^2 - 4x - 4 = 4 - 4$$
$$x^3 + x^2 - 4x - 4 = 0$$
$$(x^3 + x^2) + (-4x - 4) = 0$$
$$x^2(x + 1) - 4(x + 1) = 0$$
$$(x + 1)(x^2 - 4) = 0$$
$$(x + 1)(x^2 - 2^2) = 0$$
$$(x + 1)(x + 2)(x - 2) = 0$$

$x + 1 = 0$ *or* $x + 2 = 0$ *or* $x - 2 = 0$

$-1 \quad -1$ $\qquad$ $-2 \quad -2$ $\qquad$ $+2 \quad +2$

$x = -1$ $\qquad$ $x = -2$ $\qquad$ $x = 2$

The roots are $x = -1$, $x = -2$, and $x = 2$.

24. $2t^5 + 2t^4 - 144t^3 = 0$

$$2t^3(t^2 + t - 72) = 0$$
$$2t^3(t + 9)(t - 8) = 0$$

$2t^3 = 0$ *or* $t + 9 = 0$ *or* $t - 8 = 0$

$\dfrac{2t^3}{2} = \dfrac{0}{2}$ $\qquad$ $-9 \quad -9$ $\qquad$ $+8 \quad +8$

$t^3 = 0$ $\qquad$ $t = -9$ $\qquad$ $t = 8$

$\sqrt[3]{t^3} = \sqrt[3]{0}$

$t = 0$

The equation has roots $t = -9$, $t = 8$, and a repeated root $t = 0$.

25.

$$147s - 3s^3 = 0$$
$$3s(49 - s^2) = 0$$
$$3s(7^2 - s^2) = 0$$
$$3s(7 + s)(7 - s) = 0$$
$$3s = 0 \quad or \quad 7 + s = 0 \quad or \quad 7 - s = 0$$
$$s = 0 \qquad s = -7 \qquad 7 = s$$

The roots are $s = 0$, $s = -7$, and $s = 7$.

26.

$$4y^3 - 7y^2 + 28 = 16y$$
$$4y^3 - 7y^2 + 28 - 16y = 16y - 16y$$
$$4y^3 - 7y^2 - 16y + 28 = 0$$
$$(4y^3 - 7y^2) + (-16y + 28) = 0$$
$$y^2(4y - 7) - 4(4y - 7) = 0$$
$$(4y - 7)(y^2 - 4) = 0$$
$$(4y - 7)(y^2 - 2^2) = 0$$
$$(4y - 7)(y + 2)(y - 2) = 0$$
$$4y - 7 = 0 \quad or \quad y + 2 = 0 \quad or \quad y - 2 = 0$$
$$+7 \quad +7 \qquad -2 \quad -2 \qquad +2 \quad +2$$
$$4y = 7 \qquad y = -2 \qquad y = 2$$
$$\frac{4y}{4} = \frac{7}{4}$$
$$y = \frac{7}{4}$$

The roots are $y = \frac{7}{4}$, $y = -2$, and $y = 2$.

27. Let $y = 0$.

$$y = x^3 - 81x$$
$$0 = x^3 - 81x$$
$$0 = x(x^2 - 81)$$
$$0 = x(x^2 - 9^2)$$
$$0 = x(x + 9)(x - 9)$$
$$x = 0 \quad or \quad x + 9 = 0 \quad or \quad x - 9 = 0$$
$$\underline{-9} \quad \underline{-9} \qquad \underline{+9} \quad \underline{+9}$$
$$x = -9 \qquad x = 9$$

The x-intercepts are the roots $x = 0$, $x = -9$, and $x = 9$.

28. Let $y = 0$.

$$y = -3x^4 - 24x^3 - 45x^2$$
$$0 = -3x^4 - 24x^3 - 45x^2$$
$$0 = -3x^2(x^2 + 8x + 15)$$
$$0 = -3x^2(x + 3)(x + 5)$$
$$-3x^2 = 0 \quad or \quad x + 3 = 0 \quad or \quad x + 5 = 0$$
$$\frac{-3x^2}{-3} = \frac{0}{-3} \qquad \underline{-3} \quad \underline{-3} \qquad \underline{-5} \quad \underline{-5}$$
$$x^2 = 0 \qquad x = -3 \qquad x = -5$$
$$\sqrt{x^2} = \sqrt{0}$$
$$x = 0$$

The x-intercepts are the roots $x = -5$, $x = -3$, and the repeated root $x = 0$.

29. Let $y = 0$.

$$y = -2x^4 + 16x^3 - 32x^2$$
$$0 = -2x^4 + 16x^3 - 32x^2$$
$$0 = -2x^2(x^2 - 8x + 16)$$
$$0 = -2x^2(x^2 - 2(x)(4) + 4^2)$$
$$0 = -2x^2(x - 4)^2$$

$$-2x^2 = 0 \quad or \quad x - 4 = 0$$
$$\frac{-2x^2}{-2} = \frac{0}{-2} \qquad \underline{+4} \quad \underline{+4}$$
$$x^2 = 0 \qquad x = 4$$
$$\sqrt{x^2} = \sqrt{0}$$
$$x = 0$$

The x-intercepts are the repeated roots $x = 0$ and $x = 4$.

30. Let $y = 0$.

$$y = 4x^3 + 25x^2 - 56x$$
$$0 = 4x^3 + 25x^2 - 56x$$
$$0 = x(4x^2 + 25x - 56)$$
$$0 = x(x + 8)(4x - 7)$$

$$x = 0 \quad or \quad x + 8 = 0 \quad or \quad 4x - 7 = 0$$
$$\underline{-8} \quad \underline{-8} \qquad \underline{+7} \quad \underline{+7}$$
$$x = -8 \qquad 4x = 7$$
$$\frac{4x}{4} = \frac{7}{4}$$
$$x = \frac{7}{4}$$

The x-intercepts are the roots $x = 0$, $x = -8$, and $x = \frac{7}{4}$.