PRINCIPLES - LESSON 4A
SPECIAL EQUATIONS

Solve.

ex1) $$\frac{a+4}{3} = \frac{2a-1}{4}$$

$$4(a+4) = 3(2a-1)$$

$$4a+16 = 6a-3$$

$$-6a \qquad -6a$$

$$-2a+16 = -3$$

$$-2a = -19$$

$$\boxed{a = \frac{19}{2}}$$

What strange magic is this that allows us to cross-multiply? Which algebraic properties are we using? Why does this work?

CLEARING FRACTIONS

Solve.

ex2) $$\frac{x}{3}^{\cdot 18} + \frac{2x}{9}^{\cdot 18} - \frac{1}{6}^{\cdot 18} = 1^{\cdot 18} - \frac{5x}{2}^{\cdot 18}$$

(common denominator of 3, 9, 6, 2 = 18)

To get rid of fractions in an equation, multiply EVERY term in the equation by the common denominator of all of the fractions.

That is what we are really doing when we cross-multiply.

$$6x + 4x - 3 = 18 - 45x$$

$$10x - 3 = 18 - 45x$$

$$+45x \qquad\qquad +45x$$

$$55x - 3 = 18$$

$$55x = 21$$

$$\boxed{x = \frac{21}{55}}$$

CLEARING FRACTIONS

Solve.

ex3) $$\frac{r+1}{2} \cdot 6 + \frac{r}{3} \cdot 6 = \frac{r}{2} \cdot 6$$

(common denominator of 2, 3, 2 = 6)

$$3(r+1) + 2r = 3r$$

$$3r + 3 + 2r = 3r$$

$$5r + 3 = 3r$$

$$-5r \qquad -5r$$

$$3 = -2r$$

$$-\frac{3}{2} = r \longrightarrow \boxed{r = -\frac{3}{2}}$$

CLEARING FRACTIONS

Solve.

ex4) $\frac{2n}{5} + 3(n-1) - \frac{3}{10} = -\frac{7n}{4} + 2$

It is easier to get rid of grouping (distribute) BEFORE clearing fractions

$\frac{2n}{5}\cdot 20 + 3n\cdot 20 - 3\cdot 20 - \frac{3}{10}\cdot 20 = -\frac{7n}{4}\cdot 20 + 2\cdot 20$

(common denominator of 5, 10, 4 = 20)

$8n + 60n - 60 - 6 = -35n + 40$

$68n - 66 = -35n + 40$

$+35n \qquad\qquad +35n$

$103n - 66 = 40$

$103n = 106$

$\boxed{n = \frac{106}{103}}$

WHAT ON EARTH IS GOING ON?

Solve.

ex5) $3 + 6x = 2(3x + 1)$

$$3 + 6x = 6x + 2$$

$$-6x \qquad -6x$$

$$3 = 2$$

What happened?
The variable completely cancelled out!

But 3 does not equal 2 and it never will. There is no way to balance this scale.

$\boxed{\text{There is no solution to this equation.}}$

WHAT ON EARTH IS GOING ON?

Solve.

ex6) $2 + 6x = 2(3x + 1)$

$$2 + 6x = 6x + 2$$
$$-6x \quad -6x$$
$$2 = 2$$

Again, the variable has cancelled! But this time, we are left with a TRUE statement. 2 really does equal 2. Always. This scale will be balanced no matter what value we substitute for x.

All real numbers are solutions to this equation.

WHEN THE VARIABLE CANCELS

When the variable completely cancels out, we have to evaluate the equation that remains.

WHEN VARIABLE CANCELS OUT

Remaining equation TRUE

examples: $5 = 5$ or $-18 = -18$

Solution is ALL REAL NUMBERS

(every number will make the equation true)

Remaining equation FALSE

examples: $2 = -3$ or $9 = 0$

There is NO SOLUTION

(no number will make the equation true)