PRINCIPLES - LESSON 9B
SOLVING SYSTEMS BY SUBSTITUTION

To solve a system of equations by substitution:

1) Solve either equation for either variable.
2) Substitute that value in for the variable in the OTHER equation.

ex1) Solve by substitution:

$$\begin{cases} y = -2x + 3 \\ 3x - 2y = 8 \end{cases}$$

This equation is already solved for y.

Since $y = -2x + 3$, anywhere we see a y, we can substitute $-2x + 3$ in its place. Let's do that in the 2nd equation.

$$3x - 2y = 8$$

This equation now contains only one variable (x). We can now solve for x.

$$3x - 2(-2x + 3) = 8$$

$$3x + 4x - 6 = 8$$

$$7x - 6 = 8$$

$$7x = 14$$

$$\underline{x = 2}$$

We've now found the value of the x-coordinate of the solution to the system of equations. Since $x = 2$, we can now substitute 2 for x <u>in any equation</u> to figure out y.

$$y = -2x + 3$$

$$y = -2(2) + 3$$

$$y = -4 + 3$$

$$\underline{y = -1}$$

$$\boxed{(2, -1) \text{ is the solution to this system}}$$

SOLVING SYSTEMS OF EQUATIONS BY SUBSTITUTION

ex2) Solve by substitution:

$$\begin{cases} y = 2x + 1 \\ x + 3y = 31 \end{cases}$$

This equation is already solved for y.

Since $y = 2x + 1$, anywhere we see a y, we can substitute $2x + 1$ in its place. Let's do that in the 2nd equation.

$$x + 3y = 31$$

This equation now contains only one variable (x). We can now solve for x.

$$x + 3(2x+1) = 31$$

$$x + 6x + 3 = 31$$

$$7x + 3 = 31$$

$$7x = 28$$

$$\underline{x = 4}$$

We've now found the value of the x-coordinate of the solution to the system of equations. Since $x = 4$, we can now substitute 4 for x in any equation to figure out y.

$$y = 2x + 1$$

$$y = 2(4) + 1$$

$$y = 8 + 1$$

$$\underline{y = 9}$$

(4,9) is the solution to the system.

SOLVING SYSTEMS OF EQUATIONS BY SUBSTITUTION

ex3) Solve by substitution: $\begin{cases} 2y - 3x = -25 \\ 3y + x = 1 \end{cases}$

Neither equation is solved for one of its variables. We will have to choose an equation and a variable to solve for. Any variable will work, but choose a variable that is easy to solve for. Variables that have coefficients of 1 or -1 are easiest.

$$3y + x = 1$$
$$-3y \qquad -3y$$
$$\underline{\underline{x = 1 - 3y}}$$

I chose to solve the bottom equation for x.

This equation now contains only one variable (y). We can now solve for y.

$$2y - 3x = -25$$
$$2y - 3(1 - 3y) = -25$$
$$2y - 3 + 9y = -25$$
$$11y - 3 = -25$$
$$11y = -22$$
$$\underline{\underline{y = -2}}$$

We've now found the value of the y-coordinate of the solution to the system of equations. Since y = -2, we can now substitute -2 for y <u>in any equation</u> to figure out x.

$$x = 1 - 3y$$
$$x = 1 - 3(-2)$$
$$x = 1 + 6$$
$$\underline{\underline{x = 7}}$$

(7, -2) is the solution to the system

SOLVING SYSTEMS OF EQUATIONS BY SUBSTITUTION

ex4) Solve by substitution: $\begin{cases} 2x - y = -11 \\ 3x - 6y = 6 \end{cases}$

Neither equation is solved for one of its variables. We will have to choose an equation and a variable to solve for. Any variable will work, but choose a variable that is easy to solve for. Variables that have coefficients of 1 or -1 are easiest.

I chose to solve the top equation for y.

$$2x - y = -11$$
$$-2x \qquad\qquad -2x$$
$$-y = -2x - 11$$
$$\underline{y = 2x + 11}$$

This equation now contains only one variable (x). We can now solve for x.

$$3x - 6y = 6$$
$$3x - 6(2x + 11) = 6$$
$$3x - 12x - 66 = 6$$
$$-9x - 66 = 6$$
$$-9x = 72$$
$$\underline{x = -8}$$

We've now found the value of the x-coordinate of the solution to the system of equations. Since x = -8, we can now substitute -8 for x <u>in any equation</u> to figure out y.

$$y = 2x + 11$$
$$y = 2(-8) + 11$$
$$y = -16 + 11$$
$$\underline{y = -5}$$

$\boxed{(-8, -5) \text{ is the solution to the system}}$

SOLVING SYSTEMS OF EQUATIONS BY SUBSTITUTION

ex5) Solve by substitution:
$$\begin{cases} 25r - 10n = 100 \\ 5n + 15r = 60 \end{cases}$$

Neither equation is solved for one of its variables. We will have to choose an equation and a variable to solve for. In this equation, no term has a coefficient of 1 or -1, but every term in the bottom equation is divisible by coefficient 5.

For that reason, I chose to solve the bottom equation for n.

$$5n + 15r = 60$$
$$-15r \qquad -15r$$
$$\frac{5n}{5} = \frac{60}{5} - \frac{15r}{5}$$
$$n = 12 - 3r$$

This equation now contains only one variable (r). We can now solve for r.

$$25r - 10n = 100$$
$$25r - 10(12 - 3r) = 100$$
$$25r - 120 + 30r = 100$$
$$55r - 120 = 100$$
$$55r = 220$$
$$r = 4$$

We've now found the value of the r-coordinate of the solution to the system of equations. Since r = 4, we can now substitute 4 for r in any equation to figure out n.

$$n = 12 - 3r$$
$$n = 12 - 3(4)$$
$$n = 12 - 12$$
$$n = 0$$

If you want to write the solution as an ordered pair, write the variables as coordinates in alphabetical order.

(n, r)

$\boxed{(0, 4) \text{ is the solution to the system}}$