# PRINCIPLES - LESSON 9C
# SOLVING SYSTEMS BY ELIMINATION

**ex1) Solve by elimination:**

$$\begin{cases} x - y = -3 \\ -2x + y = -4 \end{cases}$$

Add the two equations and the y terms will cancel.

$$-1x = -7$$

$$\underline{\underline{x = 7}}$$

This allows us to solve for the variable that remains.

We've now found the value of the x-coordinate of the solution to the system of equations. Since $x = 7$, we can now substitute $7$ for $x$ <u>in any equation</u> to figure out y.

$$x - y = -3$$

$$(7) - y = -3$$

$$-y = -10$$

$$\underline{\underline{y = 10}}$$

$(7, 10)$ is the solution to the system.

# SOLVING SYSTEMS OF EQUATIONS BY ELIMINATION

**ex2) Solve by elimination:**

$$\begin{cases} 4x + 10y = 30 \\ -4x + 7y = -13 \end{cases}$$

Add the two equations and the x terms will cancel.

$$17y = 17$$

$$y = 1$$

This allows us to solve for the variable that remains.

We've now found the value of the y-coordinate of the solution to the system of equations. Since $y = 1$, we can now substitute 1 for y <u>in any equation</u> to figure out x.

$$4x + 10y = 30$$

$$4x + 10(1) = 30$$

$$4x + 10 = 30$$

$$4x = 20$$

$$x = 5$$

(5, 1) is the solution to the system.

# SOLVING SYSTEMS OF EQUATIONS BY ELIMINATION

**ex3) Solve by elimination:**

$$\begin{cases} 4x - 3y = 15 \\ 8x + 2y = -10 \end{cases}$$

$\cdot(-2)$ $\cdot(-2)$ $\cdot(-2)$

$$4x - 3y = 15 \rightarrow -8x + 6y = -30$$
$$8x + 2y = -10 \rightarrow 8x + 2y = -10$$
$$8y = -40$$
$$y = -5$$

For the elimination method to work, one of the variables MUST CANCEL when you add the two equations.

If a variable does not cancel, you must multiply one or both equations by a number that will force a variable to cancel when the equations are added.

We've now found the value of the y-coordinate of the solution to the system of equations. Since y = -5, we can now substitute -5 for y <u>in any equation</u> to figure out x.

$$8x + 2y = -10$$
$$8x + 2(-5) = -10$$
$$8x - 10 = -10$$
$$8x = 0$$
$$x = 0$$

$(0, -5)$ is the solution to the system.

# SOLVING SYSTEMS OF EQUATIONS BY ELIMINATION

**ex4) Solve by elimination:**

$$\begin{cases} 3x + 6y = 42 \\ 5x \cdot 3 - 2y \cdot 3 = 22 \cdot 3 \end{cases}$$

**I forced y to cancel by multiplying the bottom equation by 3.**

$$\begin{array}{r} 3x + 6y = 42 \\ 15x - 6y = 66 \\ \hline 18x \quad\quad = 108 \end{array}$$

$$\underline{\underline{x = 6}}$$

**We've now found the value of the x-coordinate of the solution to the system of equations. Since x = 6, we can now substitute 6 for x <u>in any equation</u> to figure out y.**

$$5x - 2y = 22$$

$$5(6) - 2y = 22$$

$$30 - 2y = 22$$

$$-2y = -8$$

$$\underline{\underline{y = 4}}$$

(6, 4) is the solution to the system.

# SOLVING SYSTEMS OF EQUATIONS BY ELIMINATION

**ex5) Solve by elimination:**

$$\begin{cases} 5x + 4y = 2 & \cdot(-3) \rightarrow -15x - 12y = -6 \\ 2x + 3y = 5 & \cdot 4 \rightarrow 8x + 12y = 20 \end{cases}$$

$$-7x = 14$$

$$\underline{x = -2}$$

To make one of the variables cancel this time, I needed to multiply BOTH equations by some numbers.

I chose to make y cancel by forcing one y coefficient to become 12 and the other to become -12.

Alternatively, we could have made x cancel by forcing one x coefficient to become 10 and the other to become -10.

It makes no difference whether x or y cancels as long as one of them does.

We've now found the value of the x-coordinate of the solution to the system of equations. Since $x = -2$, we can now substitute $-2$ for $x$ in any equation to figure out y.

$$5x + 4y = 2$$

$$5(-2) + 4y = 2$$

$$-10 + 4y = 2$$

$$4y = 12$$

$$\underline{y = 3}$$

$\boxed{(-2, 3) \text{ is the solution to the system.}}$

# SOLVING SYSTEMS OF EQUATIONS BY ELIMINATION

**Solve by elimination:**

**ex6)** $\begin{cases} 2x + 5y = 15 \\ 7y + 13 = 4x \end{cases}$

$2x + 5y = 15 \rightarrow 2x \cdot 2 + 5y \cdot 2 = 15 \cdot 2 \rightarrow 4x + 10y = 30$

$7y + 13 = 4x \rightarrow -4x + 7y = -13 \rightarrow -4x + 7y = -13$

$17y = 17$

$\underline{y = 1}$

Before we add the two equations, we have to make sure that all of the like terms line up. Add and subtract terms as needed to make this happen.

I chose to make x cancel by forcing one x coefficient to become 4 and the other to become -4. I did this by multiplying the top equation by 2.

We've now found the value of the y-coordinate of the solution to the system of equations. Since y = 1, we can now substitute 1 for y <u>in any equation</u> to figure out x.

$2x + 5y = 15$

$2x + 5(1) = 15$

$2x + 5 = 15$

$2x = 10$

$\underline{x = 5}$

(5, 1) is the solution to the system.

# SOLVING SYSTEMS OF EQUATIONS BY ELIMINATION

**ex7) Solve by elimination:**

$$\begin{cases} 3x \cdot(-3) - y \cdot(-3) = -2 \cdot(-3) \rightarrow -9x + 3y = 6 \\ 9x - 3y = 3 \rightarrow 9x - 3y = 3 \end{cases}$$

**BOTH variables cancelled and the remaining equation is FALSE. There are no values of x and y that will make this work.** → $0 = 6$ ?

$\boxed{\text{This system has no solution.}}$ (These two lines are parallel)

# SOLVING SYSTEMS OF EQUATIONS BY ELIMINATION

**ex8) Solve by elimination:**

$$\begin{cases} 3x + 2y = 5 \\ -6x - 4y = -10 \end{cases}$$

$\cdot 2 \quad \cdot 2 \quad \cdot 2$

$3x + 2y = 5 \rightarrow 6x + 4y = 10$

$-6x - 4y = -10 \rightarrow -6x - 4y = -10$

**BOTH variables cancelled and the remaining equation is TRUE.**
**There are infinite values of x and y that will make this work.** $\rightarrow$ $0 = 0$ ?

This system has infinitely many solutions. All points on the line $y = -\frac{3}{2}x + \frac{5}{2}$ are solutions to the system.

$3x + 2y = 5$

$-3x \qquad -3x$

$\frac{2y}{2} = \frac{-3x}{2} + \frac{5}{2}$

$y = -\frac{3}{2}x + \frac{5}{2}$

(These two equations are the same line.)