pp. 255-256, #7-19 all, #29

7. Substitute $17 - 4y$ for x in Equation 2 and solve for y.

$$y = x - 2$$
$$y = (17 - 4y) - 2$$
$$y = 17 - 4y - 2$$
$$y = -4y + 15$$
$$\underline{+ 4y} \quad \underline{+ 4y}$$
$$5y = 15$$
$$\frac{5y}{5} = \frac{15}{5}$$
$$y = 3$$

Substitute 3 for y in Equation 1 and solve for x.

$$x = 17 - 4y$$
$$x = 17 - 4(3)$$
$$x = 17 - 12$$
$$x = 5$$

Check

$$x = 17 - 4y \qquad y = x - 2$$
$$5 \stackrel{?}{=} 17 - 4(3) \qquad 3 \stackrel{?}{=} 5 - 2$$
$$5 \stackrel{?}{=} 17 - 12 \qquad 3 = 3 \checkmark$$
$$5 = 5 \checkmark$$

The solution is (5, 3).

8. Substitute $-3x$ for y in Equation 1 and solve for x.

$$\begin{aligned} 6x - 9 &= y \\ 6x - 9 &= -3x \\ -6x \qquad & \qquad -6x \\ -9 &= -9x \\ \frac{-9}{-9} &= \frac{-9x}{-9} \\ 1 &= x \end{aligned}$$

Substitute 1 for x in Equation 2 and solve for y.

$$\begin{aligned} y &= -3x \\ y &= -3(1) \\ y &= -3 \end{aligned}$$

Check

$$\begin{aligned} 6x - 9 &= y & \qquad y &= -3x \\ 6(1) - 9 &\stackrel{?}{=} -3 & -3 &\stackrel{?}{=} -3(1) \\ 6 - 9 &\stackrel{?}{=} -3 & -3 &= -3 \checkmark \\ -3 &= -3 \checkmark \end{aligned}$$

The solution is $(1, -3)$.

9. Substitute $16 - 4y$ for x in Equation 2 and solve for y.

$$\begin{aligned}
3x + 4y &= 8 \\
3(16 - 4y) + 4y &= 8 \\
3(16) - 3(4y) + 4y &= 8 \\
48 - 12y + 4y &= 8 \\
48 - 8y &= 8 \\
-48 \quad & \quad -48 \\
-8y &= -40 \\
\frac{-8y}{-8} &= \frac{-40}{-8} \\
y &= 5
\end{aligned}$$

Substitute 5 for y in Equation 1 and solve for x.

$$\begin{aligned}
x &= 16 - 4y \\
x &= 16 - 4(5) \\
x &= 16 - 20 \\
x &= -4
\end{aligned}$$

Check

$$\begin{aligned}
x &= 16 - 4y & \qquad 3x + 4y &= 8 \\
-4 &\stackrel{?}{=} 16 - 4(5) & \qquad 3(-4) + 4(5) &\stackrel{?}{=} 8 \\
-4 &\stackrel{?}{=} 16 - 20 & \qquad -12 + 20 &\stackrel{?}{=} 8 \\
-4 &= -4 \checkmark & \qquad 8 &= 8 \checkmark
\end{aligned}$$

The solution is $(-4, 5)$.

10. Substitute $10x - 8$ for y in Equation 1 and solve for x.

$$\begin{aligned}
-5x + 3y &= 51 \\
-5x + 3(10x - 8) &= 51 \\
-5x + 3(10x) - 3(8) &= 51 \\
-5x + 30x - 24 &= 51 \\
25x - 24 &= 51 \\
+ 24 \quad &+ 24 \\
25x &= 75 \\
\frac{25x}{25} &= \frac{75}{25} \\
x &= 3
\end{aligned}$$

Substitute 3 for x in Equation 2 and solve for y.

$$\begin{aligned}
y &= 10x - 8 \\
y &= 10(3) - 8 \\
y &= 30 - 8 \\
y &= 22
\end{aligned}$$

Check

$$\begin{aligned}
-5x + 3y &= 51 & \qquad y &= 10x - 8 \\
-5(3) + 3(22) &\stackrel{?}{=} 51 & \qquad 22 &\stackrel{?}{=} 10(3) - 8 \\
-15 + 66 &\stackrel{?}{=} 51 & \qquad 22 &\stackrel{?}{=} 30 - 8 \\
51 &= 51 \checkmark & \qquad 22 &= 22 \checkmark
\end{aligned}$$

The solution is $(3, 22)$.

11. Substitute $x + 5$ for $6y$ in Equation 1 and solve for x.

$$\begin{aligned}
-5x + 6y &= -11 \\
-5x + x + 5 &= -11 \\
-4x + 5 &= -11 \\
-4x + 5 - 5 &= -11 - 5 \\
-4x &= -16 \\
\frac{-4x}{-4} &= \frac{-16}{-4} \\
x &= 4
\end{aligned}$$

Substitute 4 for x in Equation 2 and solve for y.

$$\begin{aligned}
6y &= x + 5 \\
6y &= 4 + 5 \\
6y &= 9 \\
\frac{6y}{6} &= \frac{9}{6} \\
y &= \frac{3}{2}
\end{aligned}$$

Check

$$\begin{aligned}
-5x + 6y &= -11 & \qquad 6y &= x + 5 \\
-5(4) + 6\left(\frac{3}{2}\right) &\stackrel{?}{=} -11 & \qquad 6\left(\frac{3}{2}\right) &\stackrel{?}{=} 4 + 5 \\
-20 + 9 &\stackrel{?}{=} -11 & \qquad 9 &= 9 \checkmark \\
-11 &= -11 \checkmark
\end{aligned}$$

The solution is $\left(4, \frac{3}{2}\right)$.

12. Substitute $5y + 24$ for $8x$ in Equation 2 and solve for y.

$$\begin{aligned}
-9y &= 40 - 8x \\
-9y &= 40 - (5y + 24) \\
-9y &= 40 - 5y - 24 \\
-9y &= 5y + 16 \\
-9y + 5y &= -5y + 5y + 16 \\
-4y &= 16 \\
\frac{-4y}{-4} &= \frac{16}{-4} \\
x &= -4
\end{aligned}$$

Substitute -4 for y in Equation 2 and solve for x.

$$\begin{aligned}
8x &= 5y + 24 \\
8x &= 5(-4) + 24 \\
8x &= -20 + 24 \\
8x &= 4 \\
\frac{8x}{8} &= \frac{4}{8} \\
x &= \frac{1}{2}
\end{aligned}$$

Check

$$\begin{aligned}
8x &= 5y + 24 & -9y &= 40 - 8x \\
8\left(\frac{1}{2}\right) &\stackrel{?}{=} 5(-4) + 24 & -9(-4) &\stackrel{?}{=} 40 - 8\left(\frac{1}{2}\right) \\
4 &\stackrel{?}{=} -20 + 24 & 36 &\stackrel{?}{=} 40 - 4 \\
4 &= 4 \checkmark & 36 &= 36 \checkmark
\end{aligned}$$

The solution is $\left(\frac{1}{2}, -4\right)$.

13. Solve Equation 2 for x.

$$\begin{aligned} x - 5y &= -29 \\ x - 5y + 5y &= -29 + 5y \\ x &= 5y - 29 \end{aligned}$$

Substitute $5y - 29$ for x in Equation 1 and solve for y.

$$\begin{aligned} 2x - 3y &= -9 \\ 2(5y - 29) - 3y &= -9 \\ 10y - 58 - 3y &= -9 \\ 7y - 58 &= -9 \\ 7y - 58 + 58 &= -9 + 58 \\ 7y &= 49 \\ \frac{7y}{7} &= \frac{49}{7} \\ y &= 7 \end{aligned}$$

Substitute 7 for y in Equation 2 and solve for x.

$$\begin{aligned} x - 5y &= -29 \\ x - 5(7) &= -29 \\ x - 35 &= -29 \\ x - 35 + 35 &= -29 + 35 \\ x &= 6 \end{aligned}$$

Check

$$\begin{aligned} 2x - 3y &= -9 & \qquad x - 5y &= -29 \\ 2(6) - 3(7) &\stackrel{?}{=} -9 & \qquad 6 - 5(7) &\stackrel{?}{=} -29 \\ 12 - 21 &\stackrel{?}{=} -9 & \qquad 6 - 35 &\stackrel{?}{=} -29 \\ -9 &= -9 \checkmark & \qquad -29 &= -29 \checkmark \end{aligned}$$

The solution is (6, 7).

14. Solve Equation 2 for x.

$$x + 4y = -20$$
$$x + 4y - 4y = -20 - 4y$$
$$x = -4y - 20$$

Substitute $-4y - 20$ for x in Equation 1 and solve for y.

$$2x - y = 23$$
$$2(-4y - 20) - y = 23$$
$$-8y - 40 - y = 23$$
$$-9y - 40 = 23$$
$$-9y - 40 + 40 = 23 + 40$$
$$-9y = 63$$
$$\frac{-9y}{-9} = \frac{63}{-9}$$
$$y = -7$$

Substitute -7 for y in Equation 2 and solve for x.

$$x + 4y = -20$$
$$x + 4(-7) = -20$$
$$x - 28 = -20$$
$$x - 28 - 28 = -20 + 28$$
$$x = 8$$

Check

$$2x - y = 23 \qquad x + 4y = -20$$
$$2(8) - (-7) \stackrel{?}{=} 23 \qquad 8 + 4(-7) \stackrel{?}{=} -20$$
$$16 + 7 \stackrel{?}{=} 23 \qquad 8 - 28 \stackrel{?}{=} -20$$
$$23 = 23 \checkmark \qquad -20 = -20 \checkmark$$

The solution is $(8, -7)$.

15. Solve Equation 1 for $\frac{1}{3}x$.

$$\frac{1}{3}x + y = -1$$
$$\frac{1}{3}x + y - y = -1 - y$$
$$\frac{1}{3}x = -y - 1$$

Substitute $-y - 1$ for $\frac{1}{3}x$ in Equation 2 and solve for y.

$$\frac{1}{3}x + 8y = 13$$
$$-y - 1 + 8y = 13$$
$$7y - 1 = 13$$
$$7y - 1 + 1 = 1 + 13$$
$$7y = 14$$
$$\frac{7y}{7} = \frac{14}{7}$$
$$y = 2$$

Substitute 2 for y in Equation 1 and solve for x.

$$\frac{1}{3}x + y = -1$$
$$\frac{1}{3}x + 2 = -1$$
$$\frac{1}{3}x + 2 - 2 = -1 - 2$$
$$\frac{1}{3}x = -3$$
$$3\left(\frac{1}{3}x\right) = 3(-3)$$
$$x = -9$$

Check

$$\frac{1}{3}x + y = -1 \qquad \frac{1}{3}x + 8y = 13$$
$$\frac{1}{3}(-9) + 2 \stackrel{?}{=} -1 \qquad \frac{1}{3}(-9) + 8(2) \stackrel{?}{=} 13$$
$$-3 + 2 \stackrel{?}{=} -1 \qquad -3 + 16 \stackrel{?}{=} 13$$
$$-1 = -1 \checkmark \qquad 13 = 13 \checkmark$$

The solution is $(-9, 2)$.

16. Multiply Equation 2 by -10 and solve for $5x$.

$$\begin{aligned}
-0.5x - y &= 7.5 \\
-10(-0.5x - y) &= -10(7.5) \\
5x + 10y &= -75 \\
5x + 10y - 10y &= -75 - 10y \\
5x &= -10y - 75
\end{aligned}$$

Substitute $-10y - 75$ for $5x$ in Equation 1 and solve for y.

$$\begin{aligned}
5x + 2y &= 9 \\
-10y - 75 + 2y &= 9 \\
-8y - 75 &= 9 \\
-8y - 75 + 75 &= 9 + 75 \\
-8y &= 84 \\
\frac{-8y}{-8} &= \frac{84}{-8} \\
y &= -10.5
\end{aligned}$$

Substitute -10.5 for y in Equation 2 and solve for x.

$$\begin{aligned}
-0.5x - y &= 7.5 \\
-0.5x - (-10.5) &= 7.5 \\
-0.5x + 10.5 - 7.5 & \\
-0.5x + 10.5 - 10.5 &= 7.5 - 10.5 \\
-0.5x &= -3 \\
\frac{-0.5x}{-0.5} &= \frac{-3}{-0.5} \\
x &= 6
\end{aligned}$$

Check

$$\begin{aligned}
5x + 2y &= 9 \\
5(6) + 2(-10.5) &\stackrel{?}{=} 9 \\
30 - 21 &\stackrel{?}{=} 9 \\
9 &= 9 \checkmark
\end{aligned}$$

$$\begin{aligned}
-0.5x - y &= 7.5 \\
-0.5(6) - (-10.5) &\stackrel{?}{=} 7.5 \\
-3 + 10.5 &\stackrel{?}{=} 7.5 \\
7.5 &= 7.5 \checkmark
\end{aligned}$$

The solution is $(6, -10.5)$.

17. When substituting $3x + 4$ for $2y$, the entire expression should be subtracted by using parenthesis.

$$\begin{aligned} 7x - 2y &= 12 \\ 7x - (3x + 4) &= 12 \\ 7x - 3x - 4 &= 12 \\ 4x - 4 &= 12 \\ 4x &= 16 \\ x &= 4 \end{aligned}$$

$$\begin{aligned} 2y &= 3x + 4 \\ 2y &= 3(4) + 4 \\ 2y &= 12 + 4 \\ 2y &= 16 \\ y &= 8 \end{aligned}$$

The solution is $(4, 8)$.

18. In Step 3, 6 should be substituted for x, not for y.

Step 3
$$\begin{aligned} 3x + y &= 9 \\ 3(6) + y &= 9 \\ 18 + y &= 9 \\ \underline{-\ 18} \qquad & \underline{-\ 18} \\ y &= -9 \end{aligned}$$

19. Words

$$\boxed{\text{Number of 5-point problems}} + \boxed{\text{Number of 2-point problems}} = 38$$

$$5 \cdot \boxed{\text{Number of 5-point problems}} + 2 \cdot \boxed{\text{Number of 2-point problems}} = 100$$

Variables Let x be the number of 5-point problems on the test, and let y be the number of 2-point problems on the test.

System
$$x + y = 38 \quad \textit{Equation 1}$$
$$5x + 2y = 100 \quad \textit{Equation 2}$$

Step 1
$$x + y = 38$$
$$x - x + y = 38 - x$$
$$y = 38 - x$$

Step 2
$$5x + 2y = 100$$
$$5x + 2(38 - x) = 100$$
$$5x + 2(38) - 2(x) = 100$$
$$5x + 76 - 2x = 100$$
$$3x + 76 = 100$$
$$\underline{-76} \quad \underline{-76}$$
$$3x = 24$$
$$\frac{3x}{3} = \frac{24}{3}$$
$$x = 8$$

Step 3
$$x + y = 38$$
$$8 + y = 38$$
$$\underline{-8} \quad \underline{-8}$$
$$y = 30$$

The solution is (8, 30). So, 8 of the questions on the math test are worth 5 points, and 30 questions are worth 2 points.

29. Words $\boxed{\text{Original tens digit}} + \boxed{\text{Original ones digit}} = 11$

$$10 \cdot \boxed{\text{Original ones digit}} + \boxed{\text{Original tens digit}} =$$

$$27 + \left(10 \boxed{\text{Original tens digit}} + \boxed{\text{Original ones digit}}\right)$$

Variables Let x be the original tens digit, and let y be the original ones digit.

System
$$x + y = 11 \qquad \textit{Equation 1}$$
$$10y + x = 27 + (10x + y) \qquad \textit{Equation 2}$$

Step 1
$$x + y = 11$$
$$x - x + y = 11 - x$$
$$y = 11 - x$$

Step 2
$$10y + x = 27 + 10x + y$$
$$10(11 - x) + x = 27 + 10x + (11 - x)$$
$$10(11) - 10(x) + x = 27 + 10x + 11 - x$$
$$110 - 10x + x = 9x + 38$$
$$-9x + 110 = 9x + 38$$
$$\underline{+\,9x} \qquad \underline{+\,9x}$$
$$110 = 18x + 38$$
$$\underline{-38} \qquad \underline{-38}$$
$$72 = 18x$$
$$\frac{72}{18} = \frac{18x}{18}$$
$$4 = x$$

Step 3
$$x + y = 11$$
$$4 + y = 11$$
$$\underline{-\,4} \qquad \underline{-\,4}$$
$$y = 7$$

The solution is (4, 7). So, the original number is 47.